# Key to Tutorial 5
# Karnaugh Maps

## Exercise 1

Let us consider $N$, a 3-bit unsigned integer ($C$, $B$, $A$). $A$ is the least significant bit. Using Karnaugh maps, write down the most simplified expression of $S = f(N)$ for each of the following:

- $S = 1$ iff $N \geq 2$

| S \ BA | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| **0** | 0 | 0 | 1 | 1 |
| **1** | 1 | 1 | 1 | 1 |

**S = C + B**

- $S = 1$ iff $2 < N \leq 6$

| S \ BA | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| **0** | 0 | 0 | 1 | 0 |
| **1** | 1 | 1 | 0 | 1 |

**S = C.$\overline{B}$ + C.$\overline{A}$ + $\overline{C}$.B.A**
S = C.($\overline{B}$ + $\overline{A}$) + $\overline{C}$.B.A
S = C.($\overline{B.A}$) + $\overline{C}$.B.A
S = C$\oplus$(B.A)

- $S = 1$ iff $N = 0, 1, 2, 4, 5$ or 6

| S \ BA | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| **0** | 1 | 1 | 0 | 1 |
| **1** | 1 | 1 | 0 | 1 |

**S = $\overline{B}$ + $\overline{A}$**

- $S = 1$ iff $N = 1$ or 5 and S is undefined iff $N = 0, 2, 4$ or 6

| S \ BA | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| **0** | Φ | 1 | 0 | Φ |
| **1** | Φ | 1 | 0 | Φ |

**S = $\overline{B}$**

## Exercise 2

We want to design a circuit that performs the two's complement operation. This circuit has three inputs ($C$, $B$, $A$) and three outputs ($C'$, $B'$, $A'$). $A$ and $A'$ are the least significant bits.

1. Write down the truth table for the three outputs.

| C | B | A | C' | B' | A' |
|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 | 1 | 1 |
| 0 | 1 | 0 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 0 | 1 |
| 1 | 0 | 0 | 1 | 0 | 0 |
| 1 | 0 | 1 | 0 | 1 | 1 |
| 1 | 1 | 0 | 0 | 1 | 0 |
| 1 | 1 | 1 | 0 | 0 | 1 |

2. Write down their most simplified expressions.

**A' = A (obvious solution)**

| | | BA | | | |
|---|---|---|---|---|---|
| | **B'** | **00** | **01** | **11** | **10** |
| **C** | **0** | 0 | 1 | 0 | 1 |
| | **1** | 0 | 1 | 0 | 1 |

**B' = $\overline{B}$.A + B.$\overline{A}$**

B' = B⊕A

| | | BA | | | |
|---|---|---|---|---|---|
| | **C'** | **00** | **01** | **11** | **10** |
| **C** | **0** | 0 | 1 | 1 | 1 |
| | **1** | 1 | 0 | 0 | 0 |

**C' = $\overline{C}$.B + $\overline{C}$.A + C.$\overline{B}$.$\overline{A}$**

C' = $\overline{C}$.(B + A) + C.$\overline{B}$.$\overline{A}$

C' = $\overline{C}$.(B + A) + C.($\overline{B+A}$)

C' = C⊕(B + A)

We want to design a circuit that converts a natural binary number into a Gray code number. The gray code can be found at the end of Chapter 1. This circuit has three inputs (*C*, *B*, *A*) and three outputs (*C'*, *B'*, *A'*). *A* and *A'* are the least significant bits.

3. Write down the truth table for the three outputs.

| C | B | A | C' | B' | A' |
|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 0 | 1 |
| 0 | 1 | 0 | 0 | 1 | 1 |
| 0 | 1 | 1 | 0 | 1 | 0 |
| 1 | 0 | 0 | 1 | 1 | 0 |
| 1 | 0 | 1 | 1 | 1 | 1 |
| 1 | 1 | 0 | 1 | 0 | 1 |
| 1 | 1 | 1 | 1 | 0 | 0 |

4. Write down their most simplified expressions.

| | BA | | | |
|---|---|---|---|---|
| **A'** | **00** | **01** | **11** | **10** |
| **0** | 0 | 1 | 0 | 1 |
| **1** | 0 | 1 | 0 | 1 |

(**C** = 0 / 1)

**A' = $\overline{B}$.A + B.$\overline{A}$**

A' = B⊕A

| | BA | | | |
|---|---|---|---|---|
| **B'** | **00** | **01** | **11** | **10** |
| **0** | 0 | 0 | 1 | 1 |
| **1** | 1 | 1 | 0 | 0 |

(**C** = 0 / 1)

**B' = C.$\overline{B}$ + $\overline{C}$.B**

B' = C⊕B

**C' = C (obvious solution)**

## Exercise 3

Let us consider $N$, a 4-bit unsigned integer ($D$, $C$, $B$, $A$). $A$ is the least significant bit. Using Karnaugh maps, write down the most simplified expression of $S = f(N)$ for each of the following:

- $S = 1$ iff $N < 6$

| S (DC \ BA) | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| **00** | 1 | 1 | 1 | 1 |
| **01** | 1 | 1 | 0 | 0 |
| **11** | 0 | 0 | 0 | 0 |
| **10** | 0 | 0 | 0 | 0 |

$\mathbf{S = \overline{D}.\overline{C} + \overline{D}.\overline{B}}$

- $S = 1$ iff $N = 0, 4, 6, 8, 12$ or $14$

| S (DC \ BA) | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| **00** | 1 | 0 | 0 | 0 |
| **01** | 1 | 0 | 0 | 1 |
| **11** | 1 | 0 | 0 | 1 |
| **10** | 1 | 0 | 0 | 0 |

$\mathbf{S = C.\overline{A} + \overline{B}.\overline{A}}$

- $S = 1$ iff $N = 0, 2, 5, 7, 8, 10, 13$ or $15$

| S (DC \ BA) | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| **00** | 1 | 0 | 0 | 1 |
| **01** | 0 | 1 | 1 | 0 |
| **11** | 0 | 1 | 1 | 0 |
| **10** | 1 | 0 | 0 | 1 |

$\mathbf{S = C.A + \overline{C}.\overline{A}}$
$S = \overline{C \oplus A}$

- $S = 1$ iff $N = 2, 10, 11$ or $14$

| S (DC \ BA) | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| **00** | 0 | 0 | 0 | 1 |
| **01** | 0 | 0 | 0 | 0 |
| **11** | 0 | 0 | 0 | 1 |
| **10** | 0 | 0 | 1 | 1 |

$\mathbf{S = D.\overline{C}.B + D.B.\overline{A} + \overline{C}.B.\overline{A}}$

- $S = 1$ iff $N = 2$, 10, 11 or 14 and $S$ is undefined iff $N = 6$, 9, 13 or 15

There are two equivalent possibilities:

| **S** | **BA 00** | **01** | **11** | **10** |
|---|---|---|---|---|
| **00** | 0 | 0 | 0 | 1 |
| **01** | 0 | 0 | 0 | Φ |
| **11** | 0 | Φ | Φ | 1 |
| **10** | 0 | Φ | 1 | 1 |

**DC**

$$\mathbf{S = D.B + B.\overline{A}}$$

or

| **S** | **BA 00** | **01** | **11** | **10** |
|---|---|---|---|---|
| **00** | 0 | 0 | 0 | 1 |
| **01** | 0 | 0 | 0 | Φ |
| **11** | 0 | Φ | Φ | 1 |
| **10** | 0 | Φ | 1 | 1 |

**DC**

$$\mathbf{S = D.A + B.\overline{A}}$$